

Computationally Efficient Implementation of an Active Noise Control System Based on Partial Updates.

Pedro Ramos, Roberto Torrubia, Ana López, Ana Salinas and Enrique Masgrau.
 Communication Technologies Group (GTC). University of Zaragoza.
 E.U.P.T. Ciudad Escolar s/n, 44003, Teruel, Spain.
 pramos@unizar.es

ABSTRACT

The aim of this paper is to show the simulated and experimental results achieved with an algorithm addressed to reduce the computational costs of an Active Noise Control system. Provided that the accomplishment of causality requirements in these applications imposes strong limits in the time available for signal processing, an important reduction in the number of instructions per cycle offers the possibility of further improvements.

The algorithm employed is based on partial updates of the coefficients of the adaptive filter. By means of the controlled decimation of part of the updating process, a new parameter can be considered in the trade-off established among computational cost, convergence rate, stability and residual error.

The real implementation of the two-channel ANC system has been carried out at the front seats of a van. The specific characteristics of the main board -based on the TMS320C6701 DSP- and the A/D and D/A converters have been taken into account in order to determine the control strategy and optimize the development of the assembler code.

The results achieved indicate that a less computationally intensive algorithm can be implemented neither incrementing the residual noise nor slowing the convergence rate as long as the step size is properly chosen.

1. INTRODUCTION

The most popular adaptive algorithm used in DSP-based implementations of Active Noise Control (ANC) systems is the Filtered-x Least Mean Square (FxLMS) algorithm. Figure 1 shows the block diagram of this well known solution to attenuate acoustic noise by means of secondary sources.

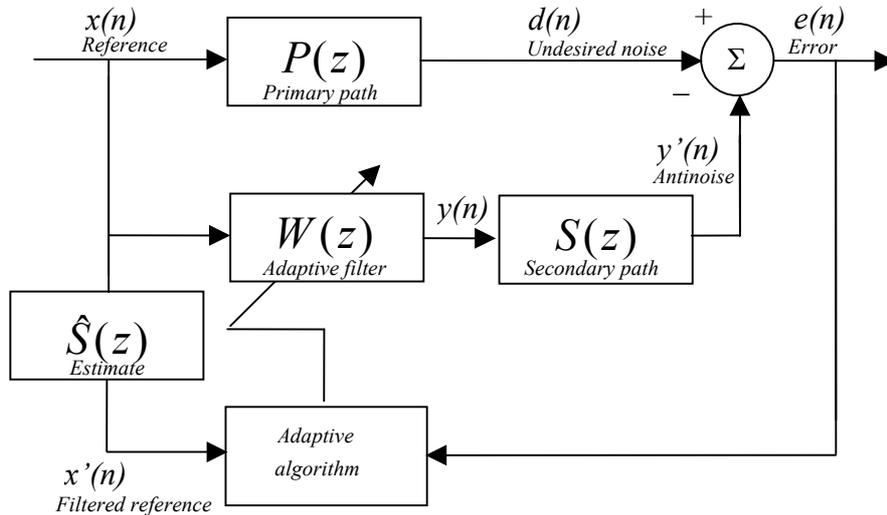


Figure 1. Active Noise Control System using a filtered-x adaptive algorithm.

This paper is focused on the saving of computational costs of the adaptive algorithm when is aimed at canceling periodic noise. The reduction in the number of instructions per iteration is carried out by means of Partial Updates (PU) of the coefficients of the adaptive filter.

The Least Mean Square (LMS) algorithm and its filtered-x version (FxLMS) have been widely used in control applications because of their ease of implementation and good performance. Nevertheless, the computational costs of these algorithms increase dramatically with the number of coefficients of the adaptive filter. Because of limitations in computational efficiency and memory capacity of low-cost DSP boards, a large number of coefficients may even impair the practical implementation of the LMS or, let alone, more complex adaptive algorithms. As an alternative to the reduction of the number of coefficients, one may choose to update only a portion of the filter coefficient vector at each time instant.

Employing decimated versions of the error and regressor signals two well known adaptive algorithms carry out the partial updating process of the filter vector [1] -in the case of an ANC system, the regressor vector corresponds to the reference signal filtered by the estimate of the secondary path (see Figure 1)-. These algorithms are, respectively, the Periodic LMS and the Sequential LMS. In this work, the attention is focused on the latter strategy, where the coefficients updates are given by:

$$w_l(n+1) = \begin{cases} w_l(n) + \mu \cdot x(n-l+1) \cdot e(n) & \dots \dots \dots (n-l+1) \bmod N = 0 \\ w_l(n) & \dots \dots \dots o.c. \end{cases} \quad 1 \leq l \leq Lw \quad [1]$$

where Lw is the length of the filter and μ the step-size of the adaptive algorithm. According to equation 1, only one of each N coefficients of the adaptive filter is updated per iteration.

On the other hand, PU algorithms suffer from one drawback: their convergence speed are reduced approximately in proportion to the filter length divided per the number of coefficients updated per iteration, that is, the decimating factor N . Therefore, the trade-off is clearly established: the more the saving in computational costs the slower the convergence rate.

This work tries to go a step further, showing that in applications of Active Control of periodic noise, the inclusion of a new parameter –that we call step-size Gain- in the traditional trade-off achieves a significant and controlled reduction in the computational costs without degrading the performance of the algorithm in terms of convergence rate or mean-square error (MSE) excess. The proposed strategy is called Filtered-x Sequential LMS algorithm with Step-Size Gain (G_μ -FxSLMS).

2. THEORETICAL ANALYSIS

A. Introduction

The traditional approach to the convergence analysis of the LMS algorithm tries to derive exact bounds on the step-size to guarantee mean and mean square convergence based upon the independence assumption, which assumes that the current set of filter coefficients is independent of the current set of data corresponding to the regressor signal [2]. This analysis has been extended to Partial Updates algorithms [1] to give the following result: the bounds upon the step-size for the Sequential LMS algorithm are the same as those for the LMS algorithm. However, this result is only valid for i.i.d (independent identically distributed) zero-mean Gaussian input signals because the analysis is only focused on the trace of the coefficient error correlation matrix. Moreover, the independence condition can not be assumed in the case of periodic input signal taking into account that the trend of the coefficients through the convergence process takes after the waveform of the periodic input signal.

To obtain a valid analysis for the case of periodic signals as input of the adaptive filter we will focus our attention on the updating process of the coefficients when the Lw -length filter is led by the PU Sequential LMS algorithm with decimating factor N .

According to equation 1, the Sequential LMS algorithm has made use of only one of each N samples of the regressor signal. Thus, it is not worth to calculate a new sample at each iteration. Calculating a new sample each N iterations would be enough.

Organizing the Lw coefficients of the filter in a matrix (equation 2), the whole updating process can be understood as the N -cyclical updating of N sub-filters of length Lw/N corresponding to each row of the matrix. The regressor signal is renewed each N iterations, that is, the N less recent values are shifted out of the valid range and a new value is introduced and will be used to update the first coefficient of each sub-filter. In equation 3, the samples of the regressor signal have been arranged below the corresponding coefficient of the adaptive filter. According to this arrangement it can be easily stated that the i -th coefficient of each sub-filter is updated by the same sample of the regressor signal. For instance, at the first iteration, the coefficients of the first sub-filter $w_1, w_{N+1}, \dots, w_{Lw-N+1}$ are updated with the same samples employed as regressor signal at the second iteration to update the second sub-filter's coefficients $w_2, w_{N+2}, \dots, w_{Lw-N+2}$.

$$Width = \frac{Fs}{Lw} \quad [6]$$

Simulations and practical experiments confirm that at these problematic frequencies the step-size Gain can not be applied at its maximum value N .

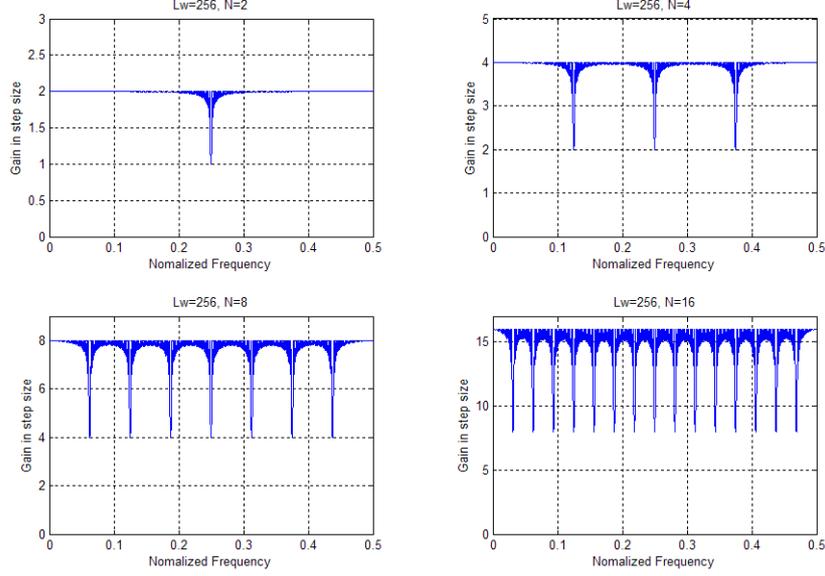


Figure 2. Step-size Gain for different decimating factors N .

The analysis made for a single tone as regressor signal may be extended to a filtered reference $x'(n)$ consisting of multiple sinusoids:

$$x'(n) = \sum_{k=1}^K C_k \cdot \cos(2 \cdot \pi \cdot k \cdot f_o \cdot n + \phi_k) \quad [7]$$

It can be proved [5] that in this more generic situation the step-size Gain can be expressed as follows:

$$G_{\mu}(k \cdot f_o, Lw, N) = \frac{1}{\lambda_{TOT,MAX}^{N>1}} = \frac{\sum_{k=1}^K C_k^2 \cdot \lambda_{k,MAX}^{N=1}(k \cdot f_o)}{\sum_{k=1}^K C_k^2 \cdot \lambda_{k,MAX}^{N>1}(k \cdot f_o)} =$$

$$= \frac{\sum_{k=1}^K C_k^2 \cdot \max \left\{ \frac{1}{4} \cdot \left(Lw \pm \frac{\sin(Lw \cdot 2 \cdot \pi \cdot k \cdot f_o)}{\sin(k \cdot 2 \cdot \pi \cdot f_o)} \right) \right\}}{\sum_{k=1}^K C_k^2 \cdot \max \left\{ \frac{1}{4} \cdot \left(\frac{Lw}{N} \pm \frac{\sin\left(\frac{Lw}{N} \cdot (N \cdot 2 \cdot \pi \cdot k \cdot f_o)\right)}{\sin(N \cdot 2 \cdot \pi \cdot k \cdot f_o)} \right) \right\}} \quad [8]$$

As happened for the single tone case, the step-size can be multiplied by N provided that the harmonics of the filtered reference were not at the notch frequencies imposed by the sampling frequency, the length of the filter and the decimating factor.

C. Algorithm Filtered-x Sequential LMS with Step-Size Gain. (G_μ -FxSLMS)

A single iteration of the Filtered-x Sequential LMS algorithm with Step-Size Gain (G_μ -FxSLMS) can be expressed as follows:

```

y(n) = w(n) · x(n)          /* Generation of antinoise */
if n mod N == 0
    x'(n) = s-hat(n) · x(n) /* Filtering with the estimate of the secondary path */
end of if
e(n) = d(n) - y(n)          /* Obtaining error signal */
for i = 1 to Lw do
    if (n-i+1) mod N == 0
        w_i(n+1) = w_i(n) + mu · G_mu(N) · e(n) · x'(n+1-i) /* Filter partial update */
    end of if
end of for

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The step-size Gain G_μ equals N if the regressor signal has no components at the notch frequencies. The main contribution of this paper is the estimate of the maximum value of this gain and its limitations.

3. CONSIDERATIONS ABOUT THE ARCHITECTURE PLATFORM

A. Digital Signal Processor and Input/Output boards

The main Digital Signal Processor board where this strategy has been implemented is the PCI/C6600, based on the DSP TMS320C6701 [4]. The Input/Output board is the PMC/Q20DS that disposes of 4 A/D and 4 D/A converters.

B. Effects of the Sampling Frequency in the Noise Level at the D/A Converters

In spite of the fact that in ANC applications the highest frequency of interest uses to be around 500 Hz –and therefore it is enough to configure the A/D converters at a sampling frequency of 1000 Hz–, we are going to increase the sampling rate as much as possible. The reason is because the experimental work with the hardware platform shows that at the A/D converters there is a noise level added to the output signals. The only way to reduce that noise level is to increase the sampling frequency. In the ANC system implemented in the van is possible the election of a maximum rate of $F_s=8000$ samples/s.

Table 1 shows the random noise level at the A/D converters for different F_s . The system is supposed to be generating a null output and, consequently, the peak to peak voltage measured corresponds to the noise.

Sampling Frequency	Peak–Peak Voltage
2000 samples/s	660 mVpp
4000 samples/s	380 mVpp
8000 samples/s	190 mVpp
24000 samples/s	100 mVpp

Table 1. Additive noise level at the A/D converters depending on the sampling frequency.

The second reason for the election of a sampling frequency 8 times higher than the minimum rate necessary to meet the Nyquist requirement can be obtained from Figure 2. The first notch at the step-size Gain G_μ appears at a frequency given for:

$$f_{first-notch} = \frac{F_s}{2 \cdot N} \quad [9]$$

Given the maximum decimating factor employed $N=8$, since the step-size Gain should be applied at the whole frequency band of interest, the first notch is above the maximum cancelable frequency to avoid instabilities in the convergence.

C. Digital Signal Processor Programming Scheme

The first reason that suggests that the employ of partial updates is more than convenient is the I/O management system supported by the hardware. But first, it is necessary to pay attention to some programming details in order to understand the structure of the assembler code: as far as the communication between the DSP and the I/O board is concerned, there are 3 different ways to carry out the transfer of information. Apart from the well-known techniques of polling and interruptions, it is also possible to execute the interruption management routine when a certain number (B) of samples, called burst, are received from the A/D converters. In so doing, the DSP has more time at its disposal - B/F_s seconds- between two consecutive executions of the routine. Moreover, the load of data usually requires much more time than any other task. By loading a larger burst the system disposes more new data spending just a bit more time in I/O operations. On the other hand, each routine execution has to deal with more than a single sample, being consequently more complicated the management of the whole burst. In our case, the simultaneous load of a burst of up to $B=8$ consecutive samples was implemented. The length of the burst can not be increased due to computational limitations. By means of a 512-length circularly indexed array the updating process of data is carried out with the $B=8$ more recent samples taken from the A/D converters. These data must be previously converted from their value in Volts to the numeric representation in the valid range of the architecture. As a result of that strategy, there are two advantages. Firstly, it is known by DSP programmers that the circular addressing is the more appropriate way to keep updated an array with an index pointing to the more recent input sample. This pointer is needed to carry out subsequent filtering of these data. The second advantage refers to the ease of implementation of a decimating process by loading the circularly indexed array with only a part of the B samples temporally saved in the buffer. Identically, taking into account the I/O routine is executed at every B th iteration, it is easy to implement a counter to execute a certain part of code every N -dependent fraction of B iterations. At this point it is necessary to remember that the FxSLMS algorithm computes some parts of the code – for instance, the updating of regressor signal- only at every N th iteration what suits well with the underlying programming scheme.

The latter reason is related to the possibility of using a second circular buffer (from 1 to N) to get at each iteration the index pointing to the first coefficient to be updated. From this initial position every N th weight must be also updated.

It is also of central importance to notice that the digital filtering is carried out taking full advantage of the DSP resources. Apart from the benefits provided by the circular addressing mentioned above, the use of techniques of pipelining programming can also reduce the number of DSP cycles required. Thus, the assembler code performs up to four successive multiplies of a discrete-time convolution in parallel. The bottle neck for a further increase in this number of simultaneous operations is imposed by the number of registers available on the DSP.

4. PERFORMANCE SIMULATION OF AN ANC SYSTEM BASED ON THE G_μ -FxSLMS ALGORITHM

A. Simulator Scheme: Transfer Functions of Primary and Secondary Paths and Spectral Analysis of Undesired Noise.

The performance of the G_μ -FxSLMS algorithm is tested when such strategy is applied to the 1x1x1 ANC system shown in the block diagram of Figure 1. The simulation was carried out in MATLAB. Figure 3.a and 3.b show the transfer functions of the primary and secondary paths $-P(z)$ and $S(z)$, respectively-. Figure 3.c shows the transfer function of the off-line estimate of secondary path. The filter modeling the primary path is a 64 order FIR filter. The secondary path is modeled -by a 4 order elliptic IIR filter- as a high pass filter whose cut-off frequency is imposed by the poor response of the loudspeakers at low frequencies. The off-line estimate of the secondary path was carried out by an adaptive FIR filter of 200 coefficients updated by the LMS algorithm as a classical problem of system identification. The sampling frequency (8000 samples/s) as well as other filter parameters were chosen in order to obtain a model quite approximate of the real implementation. Finally, Figure 3.d shows the power spectral density of the undesired noise to be cancelled, corresponding to the acoustic signal recorded inside the van when its engine runs at 840 r.p.m. The length of the adaptive filter is 256 or 512 coefficients depending on the undesired noise. At this point, it is necessary to notice an important consideration: the engine noise recorded as reference signal presents a slight increase in the instant power around 8 seconds after the start. This fact has a remarkable effect that will be later considered.

B. Results Achieved

First of all, here are compared the performance of the Filtered-x Sequential LMS algorithm for different values of the decimating term N . The step-size stays fixed (at the maximum value that guarantees convergence).

In this simulation the sign of the primary path has been changed after 10000 iterations to test the response of the adaptive algorithm to that unexpected change. The undesired noise corresponds to the engine noise whose frequency distribution was shown in Figure 3.d. Residual mean square error has been computed for the last 1000 iterations. These results appear over the graphics.

As expected, the convergence speed is reduced in inverse proportion to the number of coefficients updated per iteration.

A remarkable fact is that the previously mentioned increase in power of the reference signal causes a temporally short misadjustment on the convergence only when all the coefficients are updated at every iteration.

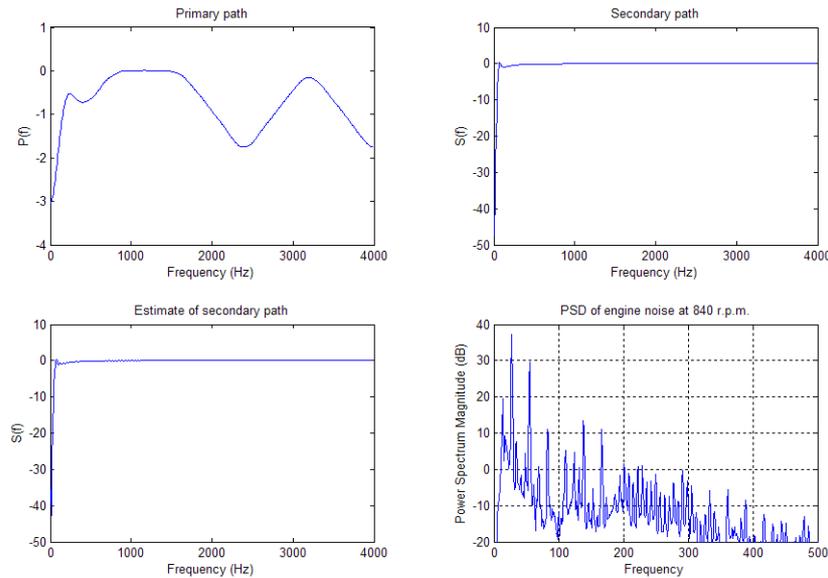


Figure 3. (a) Magnitude of primary path transfer function. (b) Magnitude of secondary path transfer function. (c) Magnitude of the estimate of secondary path transfer function. (d) Power spectral density of undesired signal.

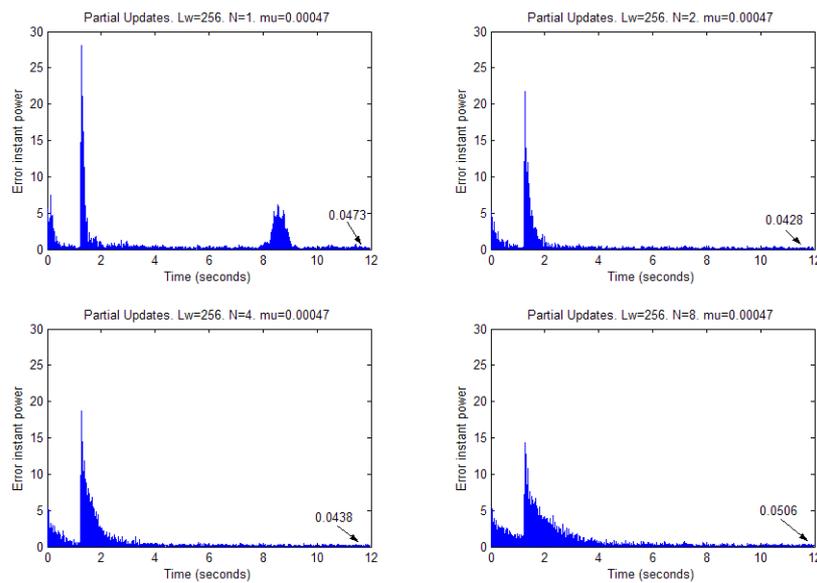


Figure 4. Convergence curve of Fx Sequential LMS algorithm with fixed step-size and with increasing parameter N .

Table 2 summarizes benefits and drawbacks of using different values of N .

Parameter	Trend	\updownarrow/\cong
Residual error	Does not change	\cong
Convergence rate	Decrease	\downarrow
Computational costs	Decrease	\downarrow
Robustness against changes in the reference power	Decrease	\downarrow

Table 2. Evolution of important parameters with increasing term N . The step-size μ remains invariable.

After having confirmed what previous researches had stated, it is time to prove that the proposed G_μ -FxSLMS algorithm achieves the same performance than the FxLMS algorithm with an important reduction in computational costs. Taking into account that the first notch in the step-size Gain appears at 2000, 1000 or 500 Hz (when $N=2, 4$ or 8 , respectively), the full strength gain $G_\mu=N$ can be applied on the step-size without any limitation because the primary noise presents all its harmonics below the lowest of these notches.

The basic step-size (for $N=1$) has been taken slightly minor than the maximum admissible to avoid instability problems at 8 seconds due to the increase in power of the reference signal.

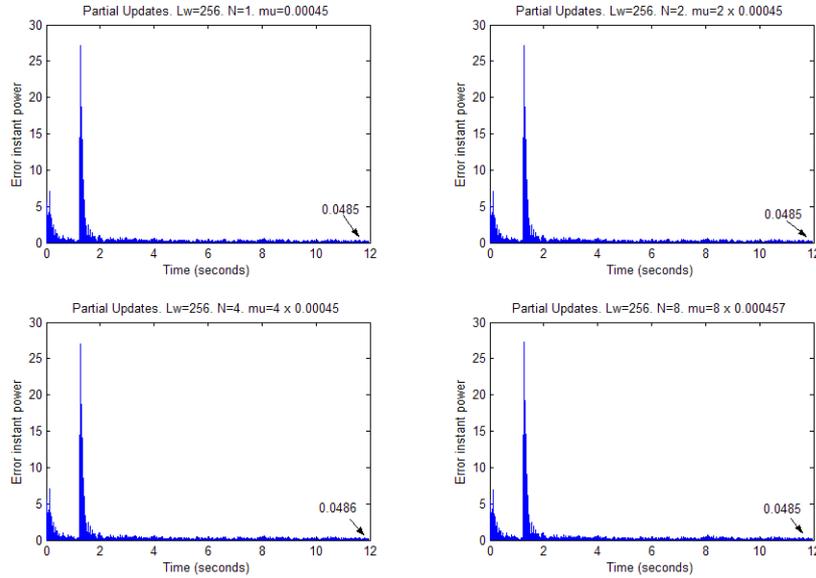


Figure 5. Convergence curve of G_μ -Fx Sequential LMS algorithm with fixed ratio $G_\mu \cdot \mu/N$ and with increasing parameter N .

Parameter	Trend	\updownarrow/\cong
Residual error	Does not change	\cong
Convergence rate	Does not change	\cong
Computational costs	Decrease	\downarrow
Robustness against changes in the reference power	Does not change (I)	\cong

Table 3. Evolution of important parameters with increasing term N . The ratio $G_\mu \cdot \mu/N$ remains invariable.

(I) Provided that the step-size is not fixed at its maximum value.

5. PRACTICAL IMPLEMENTATION OF THE G_{μ} -FxSLMS ALGORITHM IN AN ANC SYSTEM BASED ON THE DSP TMS320C6701

A. General Description of the ANC System Implemented at the Front Seats of a Van

Using the methodology outlined above, the Filtered-x Sequential LMS algorithm with Step-Size Gain was implemented. Figure 6.a shows the physical disposal of electro-acoustic elements. Figure 6.b shows the error microphone and the loudspeake acting as secondary source for the ANC system implemented at the right front seat of the van.

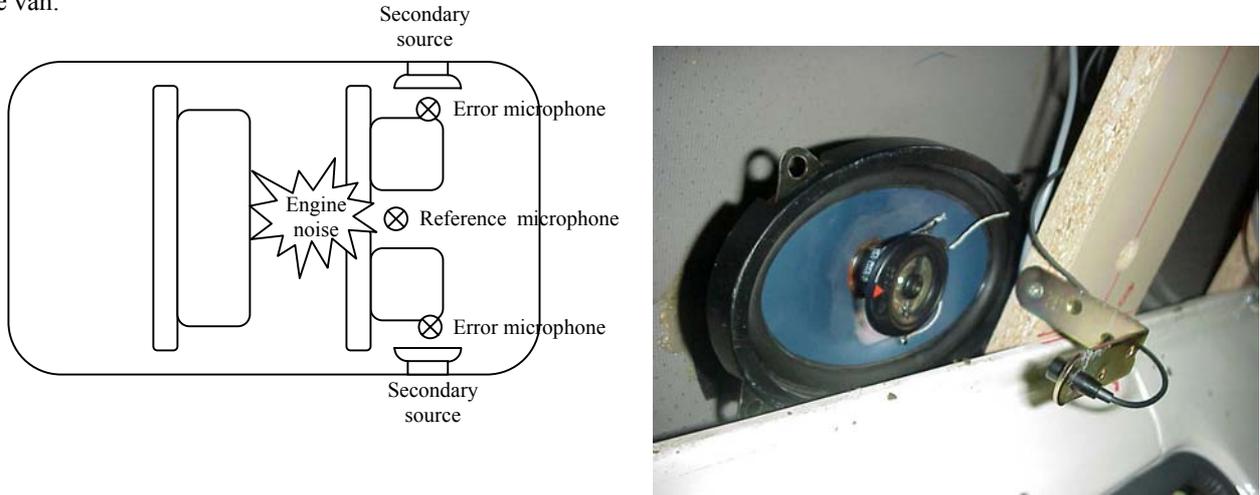


Figure 6. (a) Physical disposal of electro-acoustic elements. (b) Error microphone and secondary source.



Figure 7.(a) Control desk of the ANC system. (b) Input/Output connectors.

Figure 7 shows (a) the control desk and (b) a detail of the rack with the connectors disposed to access to the input and outputs of the architecture platform based on the DSP TMS320C6701 from Texas Instruments.

It is known [3] that the maximum degree of cancellation achievable is given by the coherence function between the reference signal and the undesired noise. Therefore, the position of the reference microphone has been optimized by means of the real-time measurement of that coherence between the acoustical signals caught at the fixed positions of the error microphones and the possible emplacements for the reference microphone. The measurements were carried out with a 5-channel acoustical analyzer PULSE 8.0.

B. Computational Costs of the Control Strategy

Although reductions in the number of operations are an indication of the computational efficiency of an algorithm, such reductions may not directly translate to a more efficient real-time DSP-based implementation on a hardware

platform. To accurately gauge such issues one must consider the freedoms and constraints that a platform impose in the real implementation, such as parallel operations, addressing modes, registers available or number of arithmetic units. In our case, the control strategy and the assembler code was developed trying to take full advantage of these issues. Table 4 summarizes the number of DSP cycles consumed by every task in which the assembler code of the two-channel Active Noise Control system can be organized. Let L_w the length of the adaptive filter, L_s the length of the estimate of the secondary path, N the decimating factor of the partial updates and B the length of the burst.

TASK	# CLOCK CYCLES
Volts to Internal representation	$10 + B \cdot 22$
Internal representation to Volts	$8 + B \cdot 24$
Set up of adaptive algorithm	$11 + B \cdot 57$
Computing outputs of adaptive filters	$B \cdot [L_w \cdot 7 + 12]$
Filtering of the reference signal	$B \cdot [(L_s \cdot 7)/N + 14]$
Partial Updates of the coefficients	$B \cdot [(L_w \cdot 6.25)/N + 21]$
TOTAL	$29 + B \cdot \left[150 + 7 \cdot L_w + \frac{1}{N} (L_s \cdot 7 + L_w \cdot 6.25) \right]$

Table 4. Number of DSP cycles per task required during the processing of an input burst of the reference signal.

Table 5 compares the number of DSP cycles required to execute the processing of a new burst of $B=8$ data with increasing number of decimating term N . The parameters chosen correspond to the real implementation of the ANC system, that is $L_w=256/512$, $L_s=200$ and $B=8$.

N	# CLOCK CYCLES	# CLOCK CYCLES
	when $L_w=256$	when $L_w=512$
1	39565	66701
2	27565	48301
4	21565	39101
8	18565	34501

Table 5. Comparison in assembler cycles required per burst.

C. Results Achieved

The parameters chosen for the G_μ -FxSLMS algorithm were $L_w=256/512$, $L_s=200$, $B=8$ and $N=G_\mu=8$. The system cancels effectively the main harmonics of the engine noise. Taking into account that the loudspeakers have a lower cut-off frequency of 60 Hz, the controller can not attenuate the components below this frequency. Besides, the ANC system finds more difficulties in the attenuation of frequency-closer harmonics. This problem can be avoid increasing the number of coefficients of the adaptive filter, for instance from $L_w=256$ to 512 coefficients (Figure 8).

In order to carry out a performance comparison of the G_μ -FxSLMS algorithm with increasing value in the decimating term N -and subsequently in the step-size Gain G_μ - it is essential to repeat the experiment with the same undesired noise. So as to avoid fluctuations in level and frequency, instead of starting the engine, we have previously recorded a signal consisting of several harmonics (100, 150, 200 and 250 Hz) engine noise. The omnidirectional source Brüel & Kjaer Omnipower 4296 placed inside the van is fed with this signal. Therefore, the comparison could be made in the same conditions. The mean square error that appears over the graphics was calculated averaging the last iterations shown. In this case, the length of the adaptive filter was 256 coefficients (Figures 9 and 10).

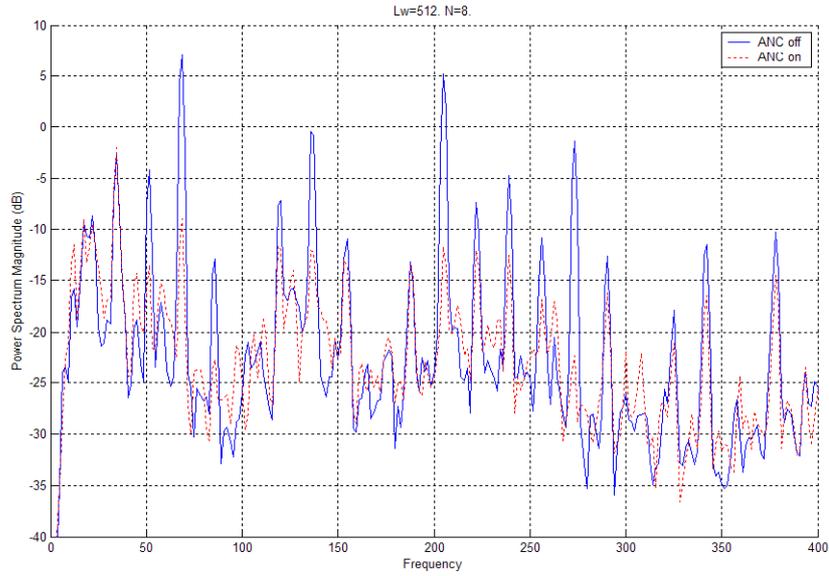


Figure 8. Practical results. Attenuation of engine noise running at 2100 r.p.m. $L_w=512$. $N=8$.

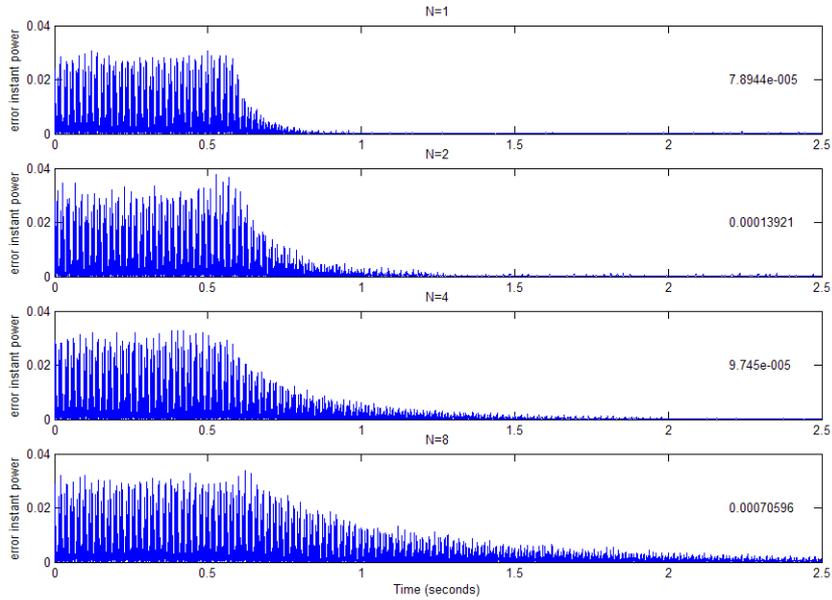


Figure 9. Practical results. Attenuation of previously recorded multi-tone noise. Fixed step-size Gain $G_\mu=1$ with increasing decimating term N . $L_w=256$.

The comparison between simulated and experimental results suits well. However, there is an important difference that it is worth to remark. Sometimes, specially at the most critic frequencies given by the notches of the step-size Gain, simulated results show that system can converges even after having been about to diverge during the first instants. When that situation occurs in the real implementation, the stability can never be recover, because the D/A converters, after having been saturated, can not produce a valid anti-noise signal unless the controller is reset. These situations must be avoided either by choosing the decimating term N and/or the sampling frequency adequately or by reducing the step-size Gain to $N/2$, with a consequent loss in the convergence rate.

6. CONCLUSIONS

In this paper, a computationally efficient strategy of Active Noise Control is proposed. This solution -called Filtered-x Sequential LMS algorithm with Step-Size Gain (G_μ -FxSLMS) - is based on the following property [5]:

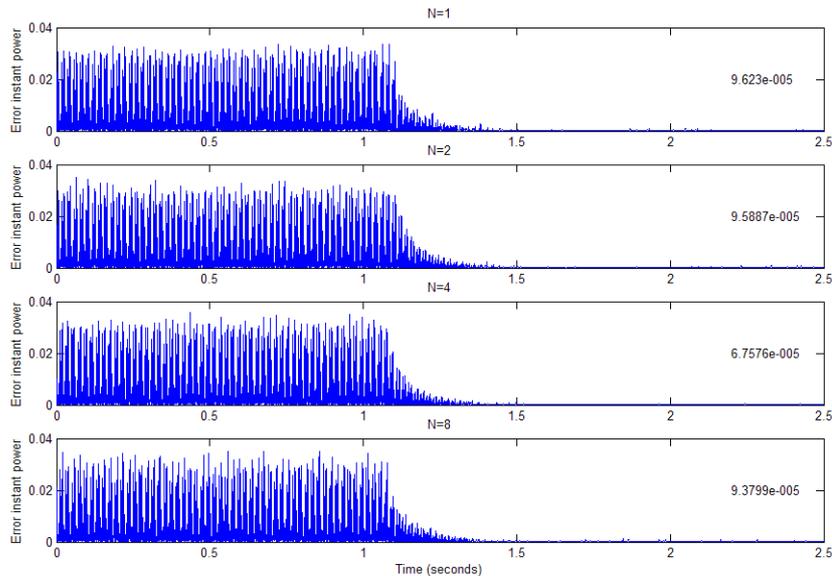


Figure 10. Practical results. Attenuation of previously recorded multi-tone noise. Variable step-size Gain $G_{\mu} = N$ with increasing decimating term N . $Lw=256$.

Let μ_a the bound on the step-size for an LMS algorithm where the regressor vector of the adaptive filter is a periodic signal consisting of several harmonics. If the regressor signal is decimated by N and the length of the adaptive filter is N times shorter, then, the bound upon the step-size is N times larger, that is $\mu_b = N \cdot \mu_a$.

This principle can be used in ANC Systems focused on the attenuation of periodic disturbances to reduce the computational costs of the control strategy by means of partial updates of the coefficients of the adaptive filter. The reduction of the computational complexity is not achieved at the expense of increasing the residual noise neither of slowing down the convergence rate.

The only condition that must be accomplished to multiply the step-size by N is that some frequencies should be avoided. These problematic frequencies correspond to the notches that appear at the step-size Gain. Their width and exact location depend on the length of the adaptive filter, the decimating term N and the sampling frequency.

Simulated and experimental results confirm the benefits of this strategy when is applied in an ANC system.

7. ACKNOWLEDGEMENTS

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